# Precise Positioning Using GPS for Category-III Aircraft Operations Using Smoothed Pseudorange Measurements

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## ABSTRACT
Currently, standard single frequency Global Positioning System (GPS) receivers provide a positioning accuracy of approximately 4-20 m. This precision can be further enhanced with dual frequency receivers which are able to provide accuracy around 1-12 m. However, these errors are quite large when it comes to safety of life applications such as aircraft landings. Differential GPS (D-GPS) allows for precise positioning using information from reference stations on the ground. Carrier phase tracking is one such D-GPS approach which allows range determination with centimeter level accuracy. However, carrier phase measurements require estimation of unknown fixed integer ambiguities before the receiver can start determining its position. Using single difference smoothed pseudorange measurements the integer ambiguities can be estimated with reasonable accuracy. This methodology brings the position error down to centimeter level which can meet the Federal Aviation Authority (FAA) regulations for Category-III (CAT-III) precision approaches. This paper examines single differenced smoothed pseudorange measurements for integer ambiguity resolution for precise positioning using carrier phase tracking. Simulation of GPS receiver performance using this methodology has been carried out and demonstrates positioning with centimeter level accuracy for the approach phase of flight. Positioning errors are also compared for dual frequency and carrier phase tracking modes.

## Categories and Subject Descriptors
J.2 [**Computer Applications**]: Physical Sciences and Engineering - *Aerospace*

## General Terms
Algorithm, Measurement, Performance, Reliability

## Keywords
Global Positioning System, Dual frequency GPS, Pseudorange, Carrier Phase, Code Phase, Integer Ambiguity



## 1. INTRODUCTION
Global Navigation Satellite Systems (GNSS) are fast becoming preferred sources of navigation information. With the advent of low cost accurate receiver technology and improved satellite coverage, the aviation industry stands to benefit highly from this technology. Developing countries such as India which are looking to expand their regional aircraft operations can do so with minimal investment in expensive ground based signaling systems such as Instrument Landing System (ILS) by relying on satellite navigation. However, commercial satellite positioning (C/A code) does not offer the precision required for safety critical applications. D-GPS promises CAT-III accuracy using existing GPS receiver along with ground based reference stations using a methodology known as carrier phase tracking. However, carrier phase measurements are biased by unknown fixed integer numbers of cycles referred to as integer ambiguities. These values must be resolved to take full advantage of the carrier phase measurements. This is referred to as the integer ambiguity resolution problem.

In this paper, first the basic principles of radio navigation using dual frequency GPS measurements are described and the simulation results of a standalone Dual-Frequency GPS receiver during flight are presented. An algorithm is presented for integer ambiguity resolution that uses single difference smoothed pseudorange measurements. This method has an advantage that it requires minimal computation compared to the conventional algorithms such as the search methods and the motion-based algorithms. It enables the determination of integer ambiguities which allow the use of phase measurements for accurate positioning of the aircraft. The use of two additional ground based GPS signal sources called integrity beacons [1] placed on the approach path to the airport is also investigated. Simulation results of a precision approach are then presented. All simulations are carried out assuming no atmospheric disturbances and an ideal Inertial Navigation System providing truth values for comparison with GPS measurements.

## 2. Positioning Using Dual Frequency Receiver
For the determination of its position on earth, the GPS receiver compares the time when the signal was sent by the satellite with the time the signal was received. From this time difference the distance between receiver and satellite can be calculated. If data from other satellites are taken into account, the present position can be calculated by trilateration (meaning the determination of a distance from three points). By means of four or more satellites,

an absolute position in a three dimensional space can be determined along with the user clock bias.

The user can estimate the pseudo range to a satellite 'i' using the following equation [2]:

$$\rho_i = c(t_{Au} - t_{Ts}) \qquad (1)$$

Where:

$$t_{Au} = t_A + b_u \qquad (2)$$

$$t_A = t_T + \frac{D}{c} + T + I \qquad (3)$$

$$t_{Ts} = t_T + B \qquad (4)$$

: Measured pseudorange to the i[th] satellite

$t_{Au}$: Measured time of arrival of signal at user

$t_{Ts}$: Value of time of transmission in message

$t_A$: True time of arrival at user

$t_T$: True time of transmission from satellite

T: Tropospheric delay [3]

I: Ionospheric delay [4]

D: Geometric range from user to satellite

B: Satellite clock error

$b_u$: User clock bias

ν: Receiver measurement noise

c: Speed of light

It can be seen from the above equations, the user needs to apply certain corrections to the measured pseudo range for the satellite clock bias which are transmitted in the GPS message. By using iono-free pseudo range measurements with a dual frequency receiver it is also possible to eliminate the iono-delay [6]. This is possible because the iono-delay is inversely proportional the frequency.

$$\rho_{if} = 2.546\rho_{L1} - 1.546\rho_{L2} \qquad (5)$$

: Pseudorange measurement using L1 carrier signal

$\rho_{L2}$: Pseudorange measurement using L1 carrier signal

$$\rho_{if} = \rho_{Ti} + c(\delta_i - \delta_R) + T + \nu \qquad (6)$$

Where is the real range from the i[th] satellite to the receiver. The pseudorange contains two primary sources of error. The two error sources are: (a) errors in the inaccurate receiver clock ($\delta_R$), called the receiver clock offset; and (b) errors in the inaccurate satellite receiving signal ($\delta_i$). Note that an important property of $\delta_R$ is that it is the same for all satellite signals and pseudoranges since it is a property of the receiver. The tropospheric delay and noise are neglected as they cannot be determined by the user. Therefore, Eqn (6) becomes:

$$\rho_{if} \cong \rho_{Ti} + c(\delta_i\ \cdot) \qquad (7)$$

$$\rho_{Ti} = \sqrt[2]{(X - X_i)^2 + (Y - Y_i)^2 + (Z - Z_i)^2} \qquad (9)$$

The satellite's position is denoted as ($X_i$, $Y_i$, $Z_i$) and the receiver's position as (X, Y, Z). The satellite position is calculated by the receiver from the ephemerides in the navigation message. The right side of Eq. (7) contains the four unknowns of X, Y, Z, and $\delta_R$. Hence, to solve for the four unknowns; a minimum of four satellites is required to yield four equations. Since Eqn. (7) is nonlinear, typically this is done using the multidimensional Newton–Raphson method and a reasonable guess of the initial receiver position [5]. The initial guess is at ($X_0$, $Y_0$, $Z_0$).

$$X = X_0 + \Delta X \;\; Y = Y_0 + \Delta Y \;\; Z = Z_0 + \qquad (8)$$

Where X, Y, Z is the true ECEF solution and $\Delta X$, $\Delta Y$ and $\Delta Z$ is the difference between the true solution and the initial guess. The initial guess at the receiver ECEF coordinates yields an initial guess for the true range. To correct the initial guess, ΔX, ΔY, ΔZ need to be determined in order to update $X_0$, $Y_0$, and $Z_0$. This is done by a linear (first-order Taylor) expansion of $\rho_{if}$ in the three spatial coordinates.

$$\rho_{if} - \rho_{Ti} - c\delta_i = \Delta X \frac{\partial \rho_i}{\partial X}\Big|_{X_0,Y_0,Z_0} + \Delta Y \frac{\partial \rho_i}{\partial Y}\Big|_{X_0,Y_0,Z_0} + \Delta Z \frac{\partial \rho_i}{\partial Z}\Big|_{X_0,Y_0,Z_0} - c\delta_R \qquad (9)$$

The solution ΔX, ΔY, ΔZ and $\delta_R$ can be found using a minimum of four equations. Defining,

$$l = \begin{bmatrix} \rho_1 & -c\delta_1 & \rho_{T1} \\ \rho_2 & -c\delta_2 & \rho_{T2} \\ \rho_3 & -c\delta_3 & \rho_{T3} \\ \rho_4 & -c\delta_4 & \rho_{T4} \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{\partial \rho_1}{\partial X}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_1}{\partial Y}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_1}{\partial Z}\Big|_{X_0,Y_0,Z_0} & 1 \\ \frac{\partial \rho_2}{\partial X}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_2}{\partial Y}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_2}{\partial Z}\Big|_{X_0,Y_0,Z_0} & 1 \\ \frac{\partial \rho_3}{\partial X}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_3}{\partial Y}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_3}{\partial Z}\Big|_{X_0,Y_0,Z_0} & 1 \\ \frac{\partial \rho_4}{\partial X}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_4}{\partial Y}\Big|_{X_0,Y_0,Z_0} & \frac{\partial \rho_4}{\partial Z}\Big|_{X_0,Y_0,Z_0} & 1 \end{bmatrix}$$

$$\delta x = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ c\delta_R \end{bmatrix}$$

$$l = A\delta x \qquad (10)$$

Using these Eqn. (10), the solution is $\delta x = A^{-1}l$. After solving for ΔX, ΔY, ΔZ and $\delta_R$ the corrected coordinates are updated to yield X, Y, and Z. This solution is, of course, an approximation so an iterative approach is required whereupon the most recent solution becomes the initial guess and the process above is repeated until the desired accuracy is obtained.

The receiver needs to take care of the fact that any random combination of 4 satellites from the visible satellites cannot be used. The receiver must check for optimal geometry by calculating the Geometric Dilution of Precision (GDOP) value for all possible combinations and selecting the combination that offers the lowest possible GDOP.

$$GDOP = \sqrt{Q^2{}_{11} + Q^2{}_{22} + Q^2{}_{33} + Q^2{}_{44}} \qquad (11)$$

Where,

$Q = (A^T A)^{-1}$

## 2.1 Results of Simulation

A point to point flight from Delhi airport to Mumbai Airport at 10,000 m altitude was simulated. There are no atmospheric disturbances to the aircraft and it is assumed to be flying at constant velocity. The error in position is plotted in Fig. 1. The error in position (magnitude of distance between true position and actual position) is between 0-8 m.

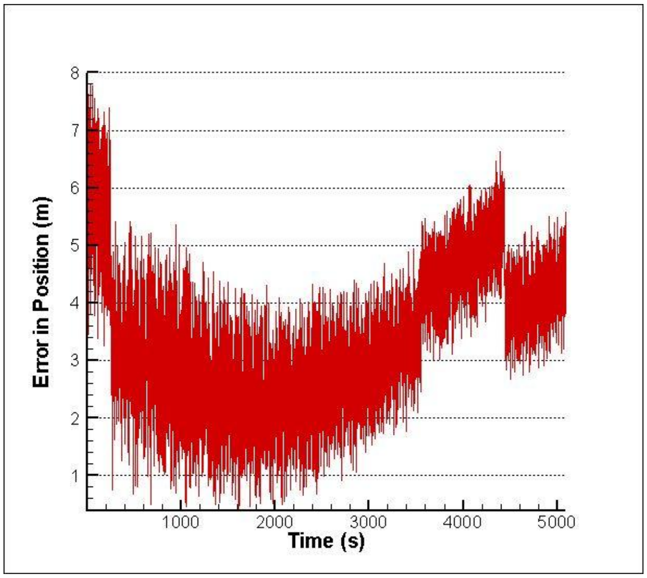


**Figure 1. Error in position (m)**

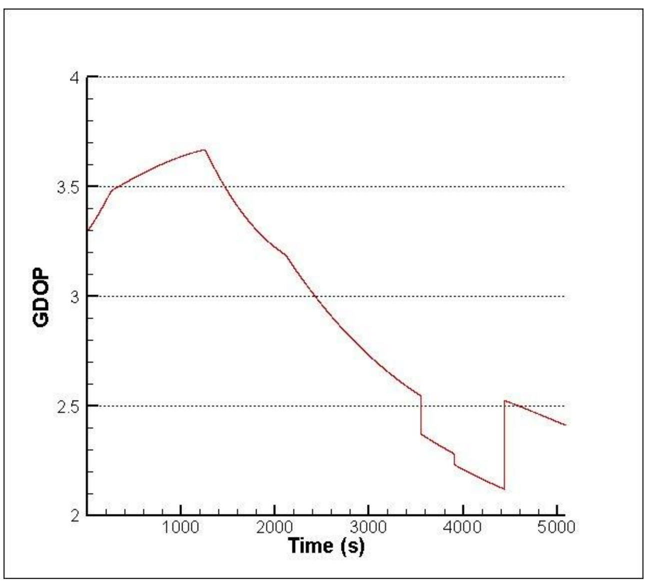


**Figure 2. Variation of GDOP during flight**

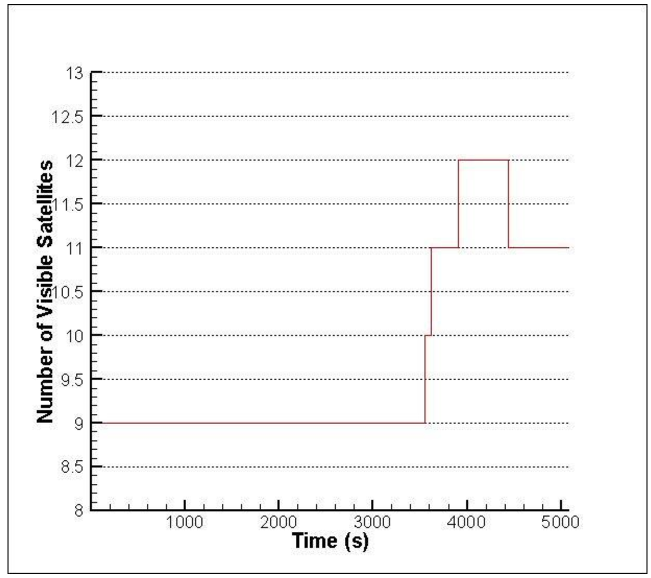


**Figure 3. Satellite visibility during flight**

Fig. 2 shows the variation of GDOP from the optimal geometry available at each instant. Fig. 3 shows the number of visible satellites during the flight.

# 3. CARRIER PHASE TRACKING

According to [6] the carrier phase can be measured with a precision of 0.01-0.05 cycle (2mm-1cm). Precise positioning which is interpreted as centimeter-level positioning requires carrier phase measurements. The carrier phase measured at the user 'u' for the satellite 'k' is:

$$\phi_u^k = r_u^k - I_u^k + T_u^k + \lambda N_u^k + \tau_u^k + \varepsilon_{\phi,u}^k \qquad (12)$$

Where,

r: Geometric range from user to satellite

$I_u^k$: Advance in phase due to ionosphere

$T_u^k$: Delay in phase due to troposphere

$N_u^k$: Integer ambiguity

$\lambda$: Carrier wavelength

$\tau_u^k$: Clock bias between user and satellite

$\varepsilon_{\varphi,u}^k$: Measurement noise

Similarly for a reference receiver 'r':

$$\phi_r^k = r_r^k - I_r^k + T_r^k + \lambda N_r^k + \tau_r^k + \varepsilon_{\phi,r}^k \qquad (13)$$

Subtracting Eqn. (13) from (12):

$$\phi_{ur}^k = r_{ur}^k - I_{ur}^k + T_{ur}^k + \lambda N_{ur}^k + \tau_{ur} + \varepsilon_{\phi,ur}^k \qquad (14)$$

Eqn. (14) gives what is called the single difference phase measurement. For short baselines i.e. when the user and reference are close it can be assumed that ionospheric and tropospheric delays are nearly the same and hence neglecting those terms:

$$\phi_{ur}^k \cong r_{ur}^k + \lambda N_{ur}^k + \tau_{ur} + \varepsilon_{\phi,ur}^k \qquad (15)$$

Thus, if the fixed integer ambiguities can be estimated, the relative vector from the reference to the user can be determined.

## 3.1 Pseudorange Smoothing[7]

The single difference code phase measurements can be calculated similar to the single difference carrier phase as:

$$\rho_{ur}^{k} \cong r_{ur}^{k} + \tau_{ur} + \varepsilon_{\rho,ur}^{k} \tag{16}$$

Subtracting Eqn. (16) from Eqn. (15) the smoothed pseudorange measurement is obtained [6]:

$$\phi_{ur}^{k} - \rho_{ur}^{k} \cong \lambda N_{ur}^{k} + \varepsilon_{\phi\rho,ur}^{k} \tag{17}$$

Thus, from Eqn. (17) it can be seen that the value of the integer ambiguity can be easily estimated at a single epoch:

$$\hat{N} = round[\lambda^{-1}(\phi_{ur}^{k} - \rho_{ur}^{k})] \tag{18}$$

: Carrier signal wavelength

These measurements are however noisy, therefore to eliminate the measurement noise, the values of the integer ambiguities over multiple epochs are averaged to obtain the final estimate. Fig. 4 shows the simulation results of integer ambiguity estimation at 100 epochs. The rounded off values are shown by the circles. The standard deviation (σ) of error in integer estimation for 100 epochs is 2.63 cycles.

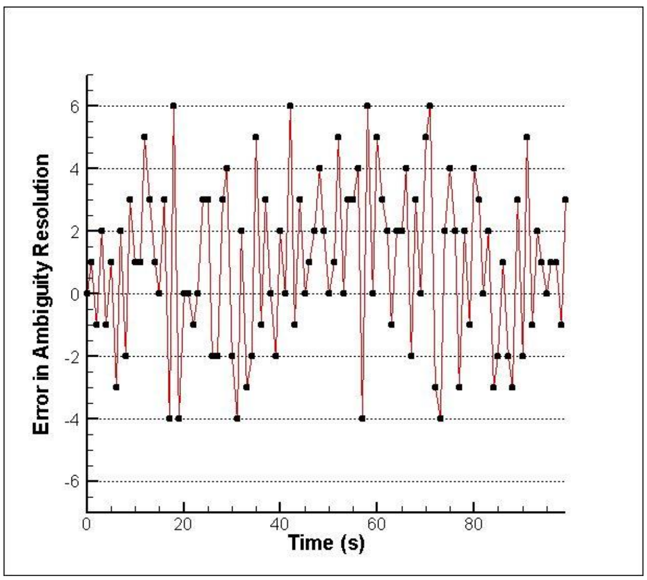


**Figure 4. Error in ambiguity resolution ( $\hat{N}$ ) for 100 epochs**

## 4. PRECISION LANDING USING INTEGRITY BEACONS [1]

A landing approach using carrier phase tracking was simulated. The airport is equipped with a precisely surveyed reference GPS receiver broadcasting its carrier and code measurements as well two integrity beacons placed approximately 16 km away from the airport on either side of the approach path. These integrity beacons provide the aircraft extra sources of measurements as well redundancy in case the user is unable to track the requisite number of satellites due to bad atmospheric conditions or Selective Availability. As the aircraft flies over these integrity beacons it starts collecting the smoothed pseudo range measurements as described in Eqn. (17). After collecting measurements over multiple epochs, the user averages the values and obtains the integer ambiguities according to Eqn. (18). Post resolution of the fixed integer ambiguities, the user plugs them back into Eqn. (15). At any epoch 't' if there are 'n' satellites visible then there are 'n+2' (due to the two integrity beacons) such equations. An approximation is made that for short baselines the line of sight vectors from the reference and user to a satellite are nearly the same. Thus for satellite 'k':

$$\phi_{ur}^{k} \cong -\vec{s}_{ur}^{\,k} \cdot \vec{x} + \lambda N_{ur}^{k} + \tau_{ur} + \varepsilon_{\phi,ur}^{k} \tag{19}$$

$\vec{x}$: Relative position of the user w.r.t the reference station

$\vec{s}_{ur}^{\,k}$: Line of sight vector to satellite

Similarly for the integrity beacon 'j':

$$\varphi_{ur}^{j} = |\vec{p_j} - \vec{x}| - |\vec{p_j}| + \lambda N_{ur}^{j} + \tau_{ur} + \varepsilon_{\phi,ur}^{k} \tag{20}$$

$p_j$: Relative vector from reference station to integrity beacon j

Given an approximate trajectory $\vec{\bar{x}}$ obtained from code based measurements, the above equations can be expressed in terms of the deviation from the approximate trajectory $\delta\vec{x} = \vec{x} - \vec{\bar{x}}$

$$\delta\phi_{ur}^{k} = \phi_{ur}^{k} + \vec{s}_{ur}^{\,k} \cdot \vec{\bar{x}} = -\vec{s}_{ur}^{\,k} \cdot \delta\vec{x} + \lambda N_{ur}^{k} + \tau_{ur} + \varepsilon_{\phi,ur}^{k} \tag{21}$$

and

$$\delta\varphi_{ur}^{j} = \varphi_{ur}^{j} - |\vec{p_j} - \vec{\bar{x}}| + |\vec{p_j}| = -\vec{e}_{ur}^{\,j} \cdot \delta\vec{x} + \lambda N_{ur}^{j} + \tau_{ur} + \varepsilon_{\phi,ur}^{k} \tag{22}$$

Where

$\vec{e}_{ur}^{\,j}$: Line of sight vector from integrity beacon j to user

Therefore, the measurements at a single epoch are stacked as:

$$\delta\phi = \begin{bmatrix} \delta\phi_{ur}^{1} \\ \vdots \\ \delta\phi_{ur}^{n} \\ \delta\varphi_{ur}^{1} \\ \delta\varphi_{ur}^{2} \end{bmatrix} \quad N = \begin{bmatrix} N_{ur}^{1} \\ \vdots \\ N_{ur}^{n} \\ N_{ur}^{IB1} \\ N_{ur}^{IB2} \end{bmatrix} \quad \hat{S} = \begin{bmatrix} -\vec{s}_{ur}^{\,1T} & 1 \\ \vdots & \vdots \\ -\vec{s}_{ur}^{\,nT} & 1 \\ -\vec{e}_{ur}^{\,1T} & 1 \\ -\vec{e}_{ur}^{\,2T} & 1 \end{bmatrix} \quad \delta x^{*} = \begin{bmatrix} \delta x \\ \tau_{ur} \end{bmatrix}$$

$$(\delta\phi - N) = \hat{S}\,\delta x^{*} \tag{23}$$

Using least squares estimation, Eqn. (23) can be solved to update the relative vector till the solution converges to a desired value.

## 4.1 Simulation Results

Fig. 5 shows the error in position as the aircraft approaches Delhi airport. For the first 120 s the aircraft is collecting the phase measurements for each epoch. After collecting this data, the navigation system calculates the integer ambiguities using the method described in section 3. Post t = 120 s, the aircraft uses carrier phase tracking (parallel to ILS where aircraft tracks glide slope upon reaching the outer marker). Fig. 5 shows that as the aircraft switches from dual frequency GPS to carrier phase tracking at t = 120s, the error in position drops drastically close to 1.4 m. As time progresses, for shorter baselines the error is gradually reducing. Fig. 6 shows the error in position during carrier phase differential GPS (CDGPS) mode.

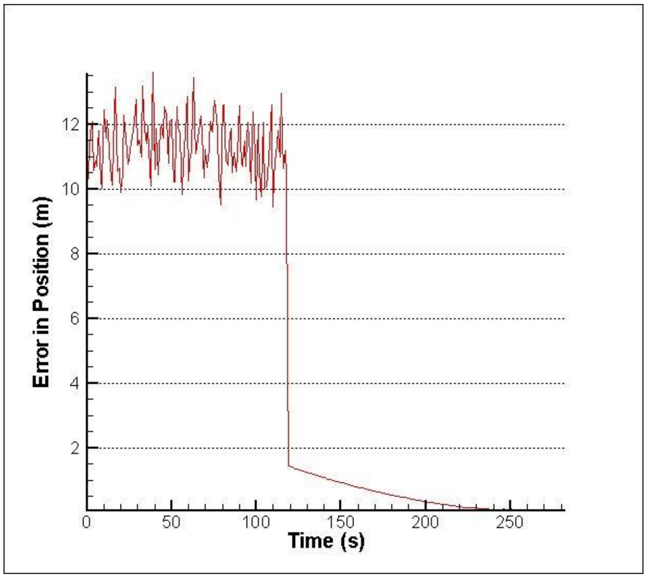


**Figure 5. Error in position during airport approach (m)**

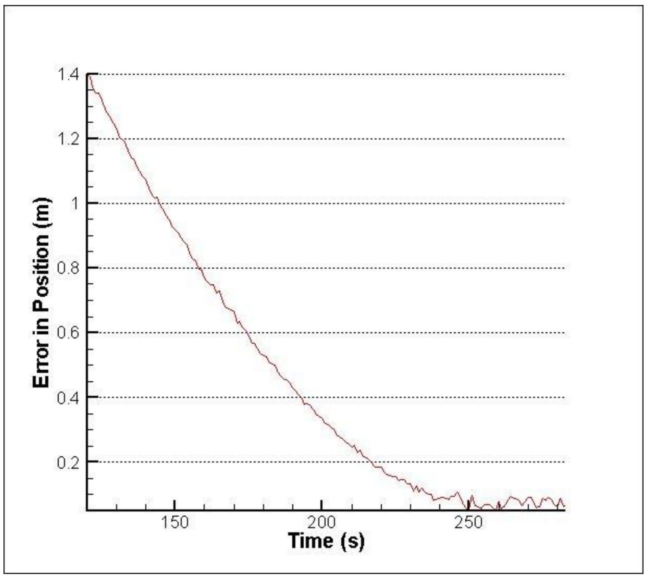


**Figure 6. Error in position during CDGPS mode**

## 5. MEETING FAA REGULATIONS

The FAA navigation accuracy requirements for precision approaches using ILS are shown in Table I.

**Table 1. FAA requirements for precision approach**

| Category | Visibility | Decision Height | Accuracy Req. 95% limits |
|---|---|---|---|
| CAT I | 800 m | 60 m | Horizontal 16.5 m<br>Vertical 3.4 m |
| CAT II | 360 m | 30 m | Horizontal 6.5 m<br>Vertical 1.6 m |
| CAT IIIa | > 210 m | < 30 m | Horizontal 4.1 m<br>Vertical 0.5 m |
| CAT IIIb | 45-210 m | < 15 m | |
| CAT IIIc | < 45 m | 0 | |

As seen from the Table I, for matching CAT-IIIa requirements for landing aircrafts cannot rely on traditional GPS positioning. Carrier phase tracking using integrity beacons on the ground is easily able to match CAT-IIIa requirements. Table II shows the navigation accuracy using carrier phase positioning. The error falls well below the range defined by the FAA.

**Table 2. Navigation accuracy using CDGPS**

| Dimension | Mean Error | Standard Deviation (σ) |
|---|---|---|
| Horizontal | 0.07 m | 0.059 m |
| Vertical | 0.25 m | 0.31 m |

## 6. Conclusions

Simulations show that a dual frequency receiver is accurate enough to provide reliable in-flight navigation, however it cannot be used for precision landing as the errors are quite large. Precise positioning using carrier phase tracking using smoothed pseudorange measurements for integer ambiguity resolution shows promising results for CAT-III approaches. One drawback is that the integer ambiguity resolution is not real time and requires that the user collect data from multiple epochs before a reliable estimate of the ambiguity can be made. This can be removed in the future with the advent of the additional L5 GPS frequency. [6] Describes a method using L1, L2 and L5 frequencies for real time estimation of the ambiguities with increased accuracy ($\sigma$ = 0.3 cycles).

## 7. FUTURE WORK

Having developed a working model of GPS the next step would be to develop a model to ingrate GPS with Inertial Navigation System INS) to allow for greater positioning accuracy (~0.4 cm). GPS/INS integration allows for the navigation system to rely on INS incase GPS signal is jammed or unavailable due to various reasons. It is of great importance for mission critical navigation such as airborne radars. The aim is to examine multiple approaches namely loosely coupled, tightly coupled and deeply integrated.

## 8. ACKNOWLEDGMENTS


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